RATIONAL EQUATIONS AND FUNCTIONS

Rational Expressions

BY RYLAN’S FIRST MATH LESSON

YOU SEE, RYLAN, ZERO IS THE ONLY NUMBER THAT ISN’T POSITIVE AND ISN’T NEGATIVE.

WHETHER YOU AGREE WITH IT OR NOT, IT’S JUST SOMETHING YOU HAVE TO ACCEPT.

AND THAT, RYLAN, IS ZERO TOLERANCE.

Name ________________________________
Algebra II

9.2 Graphing Simple Rational Functions Day One

Today I am... **graphing simple rational functions**.
I am successful today when I can... **graph simple rational functions**.
It is important for me to know/do this because... **graphs of simple rational functions can help solve real-life problems**.

1. Graph the function \( y = \frac{1}{x} \)

   \( x \)-intercept: __________
   
   \( y \)-intercept: __________
   
   vertical asymptote: __________
   
   horizontal asymptote: __________
   
   domain: __________
   
   range: __________

**Asymptote**: a line that is approached by the graph. The distance between the graph and the line approaches zero.

The graph of \( y = \frac{1}{x} \) is called a **HYPERBOLA**. It is a rational function in the form of \( y = \frac{a}{x-h} + k \).

- Hyperbolas have asymptotes of \( x = h \) and \( y = k \).
- To find the \( y \)-intercept, plug 0 in for \( x \) and solve for \( y \).
- To find the \( x \)-intercept, plug 0 in for \( y \) and solve for \( x \).
- Plot a few more points on each side of the vertical asymptote to get a graph. The graph will have two branches.

2. Graph the function \( y = \frac{3}{x-2} \)

   \( x \)-intercept: __________
   
   \( y \)-intercept: __________
   
   vertical asymptote: __________
   
   horizontal asymptote: __________
   
   domain: __________
   
   range: __________
3. Graph the function \( y = \frac{1}{x+1} + 3 \)

- \( x \)-intercept: 
- \( y \)-intercept: 
- vertical asymptote: 
- horizontal asymptote: 
- domain: 
- range: 

4. Graph the function \( y = \frac{-1}{x+4} - 3 \)

- \( x \)-intercept: 
- \( y \)-intercept: 
- vertical asymptote: 
- horizontal asymptote: 
- domain: 
- range: 

Homework: 9.2 Day One Worksheet
9.2 Graphing Simple Rational Functions Day Two

Today I am graphing simple rational functions.
I am successful today when I can graph simple rational functions.
It is important for me to know/do this because graphs of simple rational functions can help solve real-life problems.

Rational functions can also take the form \( y = \frac{ax+b}{cx+d} \). These graphs are also called hyperbolas.

- The vertical asymptote occurs at the \( x \)-value that makes the denominator zero. So, solve the denominator for zero.
- The horizontal asymptote is the line \( y = \frac{a}{c} \).
- To find the \( y \)-intercept, plug 0 in for \( x \) and solve for \( y \).
- To find the \( x \)-intercept, solve for the numerator for zero.
- Plot a few more points on each side of the vertical asymptote to get a graph. The graph will have two branches.

1. Graph the function \( y = \frac{x-2}{3x-3} \)

   \( x \)-intercept: __________

   \( y \)-intercept: __________

   vertical asymptote: __________

   horizontal asymptote: __________

   domain: __________

   range: __________

2. Graph the function \( y = \frac{2x}{x+4} \)

   \( x \)-intercept: __________

   \( y \)-intercept: __________

   vertical asymptote: __________

   horizontal asymptote: __________

   domain: __________

   range: __________
3. Graph the function \( y = \frac{-x + 2}{x - 5} \)

- \( x \)-intercept: __________
- \( y \)-intercept: __________
- vertical asymptote: __________
- horizontal asymptote: __________
- domain: __________
- range: __________

Homework: 9.2 Day Two Worksheet
Today I am graphing general rational functions. I am successful today when I can graph general rational functions. It is important for me to know/do this because graphs of general rational functions can help solve real-life problems.

**Graphs of Rational Functions**

\[ f(x) = \frac{p(x)}{q(x)} = \frac{a_nx^n + \ldots}{b_mx^m + \ldots} \]

- The \(x\)-intercepts (where graph crosses \(x\)-axis) are the real zeros of \(p(x)\)... solve numerator for zero.
- The graph of \(f\) has a **vertical asymptote** at each real zero of \(q(x)\)... solve denominator for zero.
- The graph of \(f\) has at most one **horizontal asymptote**
  
  ...if \(m < n\) (top exponent < bottom exponent), the line \(y = 0\) (\(x\)-axis) is a horizontal asymptote.
  
  ...if \(m = n\) (top exponent = bottom exponent), the line \(y = \frac{a_m}{b_n}\) is a horizontal asymptote.
  
  (Reduce the coefficients in front of the 2 variables)
  
  ...if \(m > n\) (top exponent > bottom exponent), the graph has no horizontal asymptote.

  The graph’s end behavior is the same as the graph of \(y = \frac{a_m}{b_n}x^{m-n}\).

1. \[ y = \frac{x}{x^2 - 4} \]
   - \(x\)-intercept: __________
   - \(y\)-intercept: __________
   - vertical asymptote: __________
   - horizontal asymptote: __________

2. \[ y = \frac{x^2 - 4}{x^2 + 1} \]
   - \(x\)-intercept: __________
   - \(y\)-intercept: __________
   - vertical asymptote: __________
   - horizontal asymptote: __________
3. \( y = \frac{x^2 - x - 6}{x - 2} \)

\( x \)-intercept: 

\( y \)-intercept: 

vertical asymptote: 

horizontal asymptote: 

4. \( y = \frac{8x^2}{4x^2 - 9} \)

\( x \)-intercept: 

\( y \)-intercept: 

vertical asymptote: 

horizontal asymptote: 

**Homework: 9.3 Worksheet**
Factoring Review for 9.4 Notes

Factor each expression completely.

1. \( x^2 + 6x + 8 \) 
2. \( x^2 + 9x + 8 \) 
3. \( x^2 + 2x - 8 \) 
4. \( x^2 - 7x - 8 \) 
5. \( x^2 - 9 \) 
6. \( 25x^2 - 16 \) 
7. \( x^3 - 8 \) 
8. \( 27x^3 + 64 \) 
9. \( 6x^2 - 11x + 4 \) 

Homework: Factoring Review worksheet
9.4 Multiplying and Dividing Rational Expressions

Today I am... **multiplying and dividing rational expressions**.
I am successful today when I can... **multiply and divide rational expressions**.
It is important for me to know/do this because... **rational expressions can model real-life quantities**.

**Simplifying rational expressions requires two steps:**
- Factor the numerator and denominator (if possible)
- Divide/cancel out any factors they have in common

**Simplify.**

1. \( \frac{9x^5}{5} \cdot \frac{6}{x^7} \cdot \frac{5x}{18} \)
2. \( \frac{6x^2y^3}{2x^2y^2} \cdot \frac{10x^3y^4}{18y^2} \)
3. \( \frac{9x^3y}{3x^2y^3} \cdot \frac{12x^4y^5}{27y^2} \)
4. \( \frac{x^2 - 5x - 6}{x^2 - 1} \)
5. \( \frac{2x^3 + 14x^2 - 36x}{2x^2 + 16x - 18} \)
6. \( \frac{27x^3 - 8}{3x^2 + 13x - 10} \)

**Multiply (or divide).**

7. \( \frac{x^2 - 9}{x^2 - 4x + 4} \cdot \frac{x^2 - 4}{x^2 - x - 6} \)
8. \( \frac{x - 3}{64x^3 - 1} \cdot (16x^2 + 4x + 1) \)
9. \[ \frac{3}{4x-8} \div \frac{x^2 + 3x}{x^2 + x - 6} \]

10. \[ \frac{x^5 - 4x^3}{x^2 - x - 2} \div \frac{x^5 - x^4 - 2x^3}{x^2 - 1} \]

11. \[ \frac{x}{x+3} \cdot (4x+1) \div \frac{16x^2 - 1}{x+3} \]

Homework: 9.4 Day One Worksheet
Algebra II

9.5 Day 1 Addition, Subtraction, and Complex Numbers

Today I am...adding and subtracting rational expressions and simplifying complex fractions.

I am successful today when I can...add and subtract rational expressions.

It is important for me to know/do this because...rational expressions can model real-life quantities.

***IN ORDER TO ADD/SUBTRACT FRACTIONS, YOU MUST HAVE A COMMON DENOMINATOR***

1. \( \frac{1}{18} + \frac{5}{18} \)  
2. \( \frac{4}{3x} + \frac{5}{3x} \)  
3. \( \frac{5}{12} - \frac{1}{8} \)  
4. \( \frac{2x}{x+3} - \frac{4}{x+3} \)

5. \( \frac{3x-1}{x-4} + \frac{6}{x-4} \)  
6. \( \frac{2x-1}{x+1} - \frac{x}{x+1} \)  
7. \( \frac{x}{x-2} + \frac{-8}{x^2-4} \)

8. \( \frac{6x}{3x-1} - \frac{4x}{2x+5} \)  
9. \( \frac{4}{3x^3} + \frac{x}{6x^3+3x^2} \)
10. \[
\frac{x + 1}{x^2 + 4x + 4} - \frac{2}{x^2 - 4}
\]

11. \[
\frac{x + 1}{x^2 + 6x + 9} - \frac{1}{x^2 - 9}
\]

**Homework:  9.5 Day One Worksheet**
Today I am...adding and subtracting rational expressions and simplifying complex fractions.
I am successful today when I can...add and subtract rational expressions.
It is important for me to know/do this because...rational expressions can model real-life quantities.

***IN ORDER TO ADD/SUBTRACT FRACTIONS, YOU MUST HAVE A COMMON DENOMINATOR***

1. \( \frac{1}{2} \) 
2. \( \frac{1}{2} + \frac{1}{3} \) 
3. \( \frac{x - 3}{1 + \frac{1}{x}} \)

4. \( \frac{2}{x-1} \) 
5. \( \frac{3}{x-4} \)

***Homework: 9.5 Day Two Worksheet***
9.6 Solving Rational Equations

Today I am... solving rational equations.
I am successful today when I can... solve rational equations.
It is important for me to know/do this because... rational equations can help solve real-life problems.

***Be careful when solving... you may get extraneous solutions. Plug your answer back in to the original equation to make sure it works and WATCH THAT YOU DO NOT GET DIVISION BY ZERO.***

If you have a single fraction on each side of the equation, cross multiply to solve.

1. \( \frac{x}{x-6} = \frac{1}{x-4} \)
2. \( \frac{2}{x^2 + 4x} = \frac{1}{x+4} \)

You must find a LEAST COMMON DENOMINATOR to be able to solve these types of equations.

3. \( \frac{5x}{x-2} = \frac{7 + \frac{10}{x-2}}{} \)
4. \( \frac{3}{x} - \frac{1}{2} = \frac{12}{x} \)
5. \( \frac{5}{x+1} = \frac{4 - \frac{5}{x+1}}{} \)
6. \( \frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2 - 9} \) 

7. \( \frac{3x}{x-1} + \frac{2x}{x-6} = \frac{5x^2 - 15x + 20}{x^2 - 7x + 6} \) 

Homework: 9.6 Day One Worksheet